On CPT Symmetry: Cosmological, Quantum-Gravitational and other possible violations and their phenomenology

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Abstract. I discuss various ways in which CPT symmetry may be violated, and their phenomenology in current or immediate future experimental facilities, both terrestrial and astrophysical. Specifically, I discuss first violations of CPT symmetry due to the impossibility of defining a scattering matrix as a consequence of the existence of microscopic or macroscopic space-time boundaries, such as Planck-scale Black-Hole (event) horizons, or cosmological horizons due to the presence of a (positive) cosmological constant in the Universe. Second, I discuss CPT violation due to breaking of Lorentz symmetry, which may characterize certain approaches to quantum gravity, and third, I describe models of CPT non invariance due to violations of locality of interactions. In each of the above categories I discuss experimental sensitivities. I argue that the majority of Lorentz-violating cases of CPT breaking, with minimal (linear) suppression by the Planck-mass scale, are already excluded by current experimental tests. There are however some (stringy) models which can evade these constraints.

1 CPT Breaking and the Scattering Matrix

The symmetry under the successive operations (in any order) of charge conjugation, C, parity (reflexion), P, and time reversal, T, known as CPT, is a fundamental symmetry of any *local* quantum field theory in *flat* space time, under the following assumptions [1]:

- (i) Unitarity and the proper definition of a scattering matrix,
- (ii) Lorentz Invariance and
- (iii) Locality of Interactions

In the presence of gravity, i.e. non-Minkowski,non-flat space time backgrounds, CPT symmetry may be *violated*, at least in *in its strong form*. This is indeed the case of *singular* space-time gravitational backgrounds, such as black holes, or in general space times with boundaries. The reason is that in such cases the presence of these boundaries jeopardizes requirements (i) and (ii) of the CPT theorem. In a quantum context, a Black hole evaporates due to Hawking radiation, and as such it may 'capture' for ever information on matter states passing nearby, as depicted schematically in figure 1.

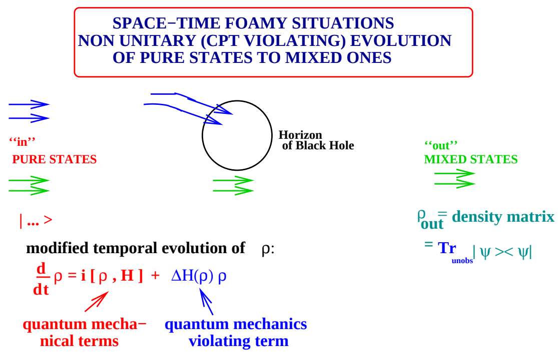

Fig. 1. When matter states pass by an evaporating Black Hole, information is lost inside the horizon. In a quantum gravity context, such black holes may appear as fluctuations of the geometry of microscopic (Planck) size, which results in the evolution of pure states to mixed for asymptotic observers, and hence in unitarity loss, and consequently CPT violation.

In such a case, one may not be able to define properly asymptotic state vectors $|\psi >$ in a quantum context, given that Planck size black hole horizons appear as fluctuations of the geometry, and hence an asymptotic observer will necessarily trace out the information captured by the horizons. This means that the out states will be necessarily described by *density matrices*, $\rho = \mathrm{Tr}_{\mathrm{unobs}}|\psi >< \psi|$. One has therefore an evolution from pure states to mixed, and unitarity is lost. The problem in defining asymptotic state vectors also implies the impossibility of defining a proper scattering S-matrix, since the latter connects by definition "in" and "out" state vectors: $|OUT >= S|IN >$. Instead, one can only define a Hawking \$-matrix [2], which connects IN and OUT mixed states described by *density matrices*:

$$\rho_{OUT} = \$\rho_{IN} \tag{1}$$

where $\$ \neq SS^\dagger$, with $S = e^{iHt}$ the S-matrix, and H the Hamiltonian of the matter subsystem. The \$-matrix *has no inverse*, as a consequence of the loss of information encountered in the problem.

This, in turn, results [3] in the impossibility of defining a proper CPT operator for such a system, and hence CPT symmetry is violated in its strong form. The proof is elementary but instructive, and hence we give it here for completeness. Consider an initial ($t \to -\infty$) density matrix, ρ_{IN}, which may or may not correspond to a pure state. Let ρ_{OUT} be the corresponding OUT state ($t \to +\infty$), which is definitely a mixed state in the case at hand (c.f. figure 1). Assume that there exists a unitary, invertible quantum-mechanical CPT operator Θ :

$$\begin{aligned} &\Theta\rho_{IN} = \overline{\rho}_{OUT} \ , \qquad \rho_{OUT} = \Theta^{-1}\overline{\rho}_{IN} \\ &\$\overline{\rho}_{IN} = \overline{\rho}_{OUT} \end{aligned} \tag{2}$$

From (1), (2) we may write: $\$\Theta^{-1}\rho_{OUT} = \Theta\rho_{IN}$, from which: $\Theta^{-1}\$\Theta^{-1}\rho_{OUT} = \rho_{IN}$.However, from (1) we can rewrite this equation in the form:

$$\Theta^{-1}\$\Theta^{-1}\$\rho_{IN} = \rho_{IN} \tag{3}$$

which implies the existence of an inverse $\Theta^{-1}\$\Theta^{-1}$ of the \$-matrix, which as explained above does not exist as a result of the information loss in the problem. Thus, we conclude that in the black hole case of figure 1, or any other case of an open quantum mechanical system with unitarity (information) loss, a strong form of CPT invariance *cannot hold* [3].

2 Cosmological Constant, \$-matrix and CPT Breaking

The conclusions of the previous section may be extended to cases of interest in astrophysics, involving space-time boundaries across which (quantum) information is lost. Such a case is that of a Robertson-Walker expanding Universe with a positive cosmological *constant* $\Lambda > 0$. In such a Universe the density of matter scales with the scale factor $a(t)$ as: $\rho \sim a^{-3}$, while the presence of a cosmological constant results in a "vacuum energy density" ρ_Λ that remains *constant*, and hence eventually *dominates* the expansion of the Universe, forcing the latter to enter a (de Sitter) phase of exponential expansion, $a(t \to \infty) \sim e^{\sqrt{\frac{\Lambda}{3}}t}$, and hence *eternal* acceleration. In such a case there is a *cosmic* horizon, i.e. a surface beyond which a cosmological observer in our Universe cannot see, given that the light takes infinite time to traverse the (finite) distance corresponding to the horizon radius δ :

$$\delta = \int_{t_0}^{\infty} = \frac{cdt}{a(t)} < \infty \tag{4}$$

The presence of a cosmic horizon makes the case of an eternally accelerating Universe with a (positive) cosmological constant Λ somewhat similar to the Black Hole case of figure 1. The important difference is that in the Λ-Universe the observer lives *inside* the cosmic (Hubble) horizon, in contrast to the Black Hole case, where the observer lives*outside* the event horizon. Nevertheless, in *both* cases, one cannot define proper asymptotic states, in the sense of information loss across the space-time boundary , and hence the \$ matrix is non factorisable: an S-matrix cannot be defined for the problem.According to the discussion in the previous section, therefore, one expects [3] a breaking of CPT symmetry in its strong form. IN this case, we call this type of breaking *Cosmological*, to distinguish its global nature from the microscopic black hole case of figure 1, which we refer to as *local quantum gravity* type of CPT breaking.

It must be mentioned at this stage that current astrophysical evidence (either by direct measurements of the acceleration of the Universe using supernovae type -Ia data [4] (c.f. left and middle figures 2) or indirect evidence from Cosmic Microwave Background (CMB) anisotropies [5] (c.f. right figure 2)) points towards a *dark* energy component of the Universe, which covers 73% of its energy

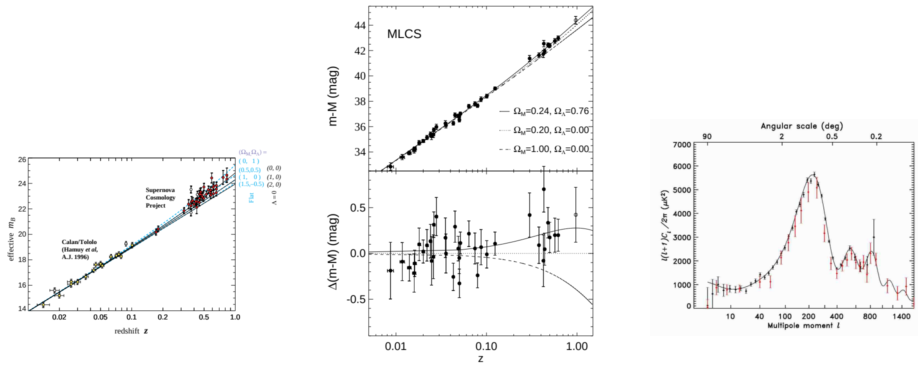

Fig. 2. Left and Middle Figures: Supernovae Ia data on the current acceleration of the Universe. Right Figure: Cosmic Microwave Background Data also point towards a current acceleration in the sense of a non-zero vacuum energy density of the Universe, covering 73% of its energy content.

content. Best-fit models to the current data are consistent with a Cosmological Constant Λ-Universe [4,6].

It is physically interesting, therefore, to ask what happens with CPT invariance of a quantum field theory when placed in such Universes (global gravitational backgrounds). From the previous arguments [3] one expects Cosmological Breaking of the CPT theorem. An immediate question concerns the *order of magnitude* of such a breaking. Certainly it is expected to be proportional to the cosmological constant Λ, but since the latter is a dimensionful quantity the order of the associated CPT violation should depend on the specific context considered. The natural framework will be to consider the time evolution of the density matrix ρ_m of matter in a Λ-Universe. The analogy with open system quantum mechanics, as well as the Black Hole case (c.f. fig.1), implies the following modified evolution equation:

$$\partial_t \rho_m = i[\rho_m, H] + \delta H(\rho_m) \tag{5}$$

where H is the Hamiltonian of the matter subsystem, and $\delta H(\rho_m)$ is in general non linear, and expresses the interaction of the matter subsystem with the "environment". In the cosmological case this environment consists of long-wavelength modes (or parts thereof) which lie outside the Hubble cosmic horizon (4). To leading order one may assume approximately that terms linear in ρ_m in $\delta H(\rho_m)$ determine to a satisfactory degree the low-energy physics, accessible to particle physics experiments. Then, linear evolutions of the type (5) may be restricted according to the Lindblad form [7]:

$$\partial_t \rho_m = i[\rho_m, H] + \sum_j a_j \rho_m a_j^\dagger - \frac{1}{2}\left(a_j^\dagger a_j \rho + \rho a_j^\dagger a_j\right) \tag{6}$$

with a_j,$a_j^\dagger$ operators expressing the interaction with the environment (the set may even be infinite). The form (6) is valid upon some reasonable assumptions

about positivity of ρ_m, conservation of its trace $\mathrm{Tr}\rho_m$=const, *etc.* Such assumptions may not be valid in some models of quantum gravity, but one may be confident that they at least characterize the cosmological constant Universe under consideration. To adapt (6) in a cosmological context one needs a specific and detailed model.

3 Non-Critical Strings, Cosmological Constant and the order of CPT Violation

One such model, to which we shall restrict our attention in this review, is provided within the framework of the so-called non-critical (Liouville) string theory [8], which - as argued in [9] - is an appropriate framework for describing *non-equilibrium* string theories. The immediate question, of course, is whether the Λ-Universe is a non equilibrium system.

The answer to such a question depends on the definition of "equilibrium" in string theory, which should not be confused with the 'thermal' one. In the first quantized version of strings we define [9] as 'equilibrium theories' the stringy σ-models whose target-space backgrounds correspond to *conformal* world-sheet couplings, in other words have vanishing world-sheet renormalization-group (RG) β-functions [1]:

$$\beta^i = \sum_{\{i_m\}} C^i_{i_1 \dots i_m} g^{i_1} \dots g^{i_m} = 0 \tag{7}$$

where $C_{i_1 \dots i_m}$ correspond to correlation functions of vertex operators of the σ-model, and g^i are σ-model couplings/target-space background fields. The theory space indices i_j are raised and lowered by a specific metric to be discussed below. Such equations determine specific (conformal) string backgrounds. As target space equations, they can be expanded in configuration space in powers of $\alpha' = \ell_s^2 = 1/M_s^2$, where M_s the string mass scale. This is a free parameter in string theory, and, thus, may be different from the four-dimensional Planck mass $M_P \sim 10^{19}$ GeV.

To lowest order in α', in the case of a gravitational background only, the respective β-function is just the Ricci tensor:

$$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu} \tag{8}$$

where μ, ν are space time indices.Thus, if other backgrounds are ignored, the condition (7) implies Ricci flat backgrounds. Such backgrounds are solutions of

[1] Actually, diffeomorphism invariance of the target space requires [10] that the β^i in (7) are the so-called Weyl-anomaly coefficients and not simply the RG β-functions $\beta_{RG} = dg^i/d\mathrm{ln}\mu$, with $\mathrm{ln}\mu$ a world-sheet scale. One has: $\beta^i = \beta^i_{RG} + \delta g^i$ where δg^i correspond to variations of the background fields g^i under target-space diffeomorphisms. For instance, for graviton backgrounds one has to $\mathcal{O}(\alpha')$, with α' the Regge slope: $\beta^G_{\mu\nu} = \alpha' R_{\mu\nu} + \nabla_{(\mu}\partial_{\nu)} W[g]$, where $W[g]$ is a scalar function of the backgrounds. To $\mathcal{O}(\alpha')$, $W[g]$ is the dilaton Φ.

Einstein equations, which thus reconciles General Relativity with stringy Conformal Invariance, at least at the level of equations of motion in target space [10]. The equivalence is more general, given that for arbitrary backgrounds $\{g^i\}$ of a stringy σ-model one can prove [11] a *gradient flow* property of the β^i-functions:

$$\beta^i = \mathcal{G}^{ij}\frac{\delta S[g]}{\delta g^j}~, \qquad \mathcal{G}^{ij} \equiv z^2\bar{z}^2 < V^i V^j >_g \tag{9}$$

where $z, \bar{z}$ are (Euclidean) world-sheet coordinates, $\mathcal{G}^{ij}$ is the Zamolodchikov (inverse) metric in theory space $\{g^i\}$, and $S[g]$ is a target-space diffeomorphism invariant functional of g, playing the rôle of an effective action. Since, at least perturbatively in (weak) couplings g^i, the function $\mathcal{G}^{ij}$ is invertible, one concludes that the conformal invariance conditions (7) are, at least perturbatively in g^i, *equivalent* to on-shell equations of motion $\frac{\delta S[g]}{\delta g^i} = 0$. When such on-shell conditions are satisfied we speak of 'equilibrium points' in string theory space.

The issue of *non-critical* strings arises in connexion with non-vanishing $\beta^i \ne 0$, which in view of the above considerations also implies off shell backgrounds in the sense of $\frac{\delta S[g]}{\delta g^i} \ne 0$. This is what we consider [9] as a *non-equilibrium* case in string theory.

In the latter case, conformal invariance of the σ-model may be *restored* by the introduction of an *extra* σ-model coordinate, the Liouville field [8], $\varphi(z,\bar{z})$, the zero-mode of which may be viewed [9] as the world-sheet RG scale.The important point is that upon Liouville dressing of the non-conformal vertex operators [8], the stringy σ-model in the extended ((D+1)-dimensional) target space (X^μ, φ), $\mu = 0, \dots, D-1$, is *conformal*.

A detailed analysis [8], shows that near a fixed point in theory space $\{g^[\}$ (where σ-model perturbation theory is valid) the Liouville dressed [8] couplings $\lambda^i(\varphi, g^j)$ 'obey the following equation:

$$\lambda^{i\,\prime\prime} + Q[\lambda^j,\varphi]\lambda^{i\,\prime} = -\beta^i(\lambda^j) \tag{10}$$

where the prime denotes differentiation with respect to the world-sheet zero mode of φ, $\beta^i(\lambda^j)$ is the world-sheet Weyl anomaly coefficient but with g^i replaced by the Liouville dressed couplings λ^j, and the minus sign in front of it is valid for *supercritical* string theories, i.e. for deformed σ-models with the running central charge $C[g] > 9$ (for superstrings), where we shall concentrate on for the purposes of this review. In this case the Liouville mode has a *time-like* signature [12], and we interpret it as target time [9].

A solution to order g^2 of (10) reads:

$$\lambda^i(g^j,\varphi) = e^{-\alpha^i\varphi}g^i + C^i_{jk}g^j g^k e^{-\alpha^i\varphi}\varphi + \mathcal{O}(g^3) \tag{11}$$

with C^i_{jk} the O.P.E. appearing in the β-functions (c.f. (7)). The quantities α^i are the gravitational anomalous dimensions [8], which are defined as follows: if we expand $Q^2[\lambda] = Q_*^2 + \mathcal{O}(g^2)$, where Q_*^2 =const, then α^i satisfy: $\alpha^i(\alpha^i + Q_*) = -(\Delta_i - 2)$, with Δ_i the conformal dimension of the vertex operator corresponding

to the coupling g^i. Note that $\Delta_i - 2$ is the anomalous dimension of this coupling under quantum (world-sheet) corrections.

Let us now consider the Λ-Universe as a *non-conformal* gravitational background of a string [9]. The background is non-conformal since to $\mathcal{O}(\alpha')$ the relevant Einstein equation is:

$$\alpha' R_{\mu\nu} = (\alpha')^2 \Lambda g_{\mu\nu} \neq 0 \; ; \qquad \Lambda > 0 \; , \tag{12}$$

from which we observe that the relevant σ-model Weyl anomaly coefficient β^G is non zero (c.f. (8)).

The conventional framework in string theory is to interpret such non-vanishing contributions to the β-functions as arising from higher world-sheet topologies, e.g. *dilaton tadpoles*, which contribute to the ultraviolet world-sheet divergences and thus need regularization. Such a regularization can be taken care of by adding appropriate counterterms at a tree level in the σ-model, which are such so as to cancel the higher-genus infinities [13].

The problem with such an approach is the convergence of the higher-genus surface resummation, as well as the fact that if this were the mechanism for generating a cosmological constant in string theory, the resulting value should be expected on generic grounds to be of order Planck (in appropriate units). It would be very hard to reconcile such a mechanism with the smallness of the currently observed cosmological constant [6].

In [9] we have proposed an alternative approach, which allows for *relaxation* mechanisms to be employed in string theory, based on the above-described Liouville string approach. According to the latter, we may view the Λ-Universe as a non-critical σ-model which needs Liouville dressing. The origin of the non-criticality in this approach is an issue that can be dealt with in the context of detailed models [14,15], which we shall not describe here. For our purposes we only mention that departure from conformality may be induced, for instance, by cosmically *catastrophic* events, such as the collision of two *brane* worlds [15]. The non-critical σ-model in such a case represents stringy matter excitations on the brane corresponding to the observable world, and the initial central charge deficit is nothing other than the effective potential between the two branes when they are closed to each other (soon after the collision). The latter can then be computed by perturbative methods via the exchange of pairs of open strings stretched between the branes.

We note at this stage that the advantage of this approach is that now higher genus world-sheet or other effects may generate Planck size cosmological constant, which, however, upon appropriate Liouville dressing results in *relaxing* to zero vacuum energy density, once the Liouville mode is *identified* with the cosmological *target time* [9].

This identification constraints the dynamics of the $(D+1)$-dimensional space time on a D-dimensional hypersurface. It is important to notice that in cases of physical interest [15] such a constraint is obtained *dynamically*, as corresponding to *minimization* of an appropriate effective potential in the D-dimensional target space.

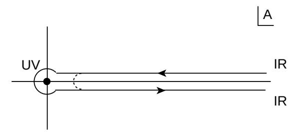

Fig. 3. Steepest-descent contour for integrating the Liouville zero mode φ in a non-critical σ-model. The curve lies in the complexified world-sheet area A plane. Upon the identification of the Liouville mode with the target time this contour becomes similar to closed time like paths in open (non-equilibrium) field theories, and leads to non factorisable \$-matrices, $\$ \neq SS^{\dagger}$, due to world-sheet ultraviolet infinities (in the region $A \to 0$).

Upon this identification, one can show [9] that the time evolution of a matter density matrix ρ_m, propagating in such non-critical string backgrounds,has the form:

$$\dot{\rho}_m(\lambda, t) = i[\rho_m, \mathcal{H}] + : \beta^i(\lambda)\mathcal{G}_{ij}[\lambda^j, \rho_m] : \tag{13}$$

where $\mathcal{H}$ is the matter low-energy Hamiltonian, and the $: \ldots :$ denote appropriate normal ordering of the quantum operators λ^i [2]. In this sense, one may view the various partitions of the operator $\beta^i\mathcal{G}_{ij}$, which is expanded in powers of λ^i, as expressing the various Lindblad "environmental" operators for the Liouville string. The presence of the environment is manifested through the non-vanishing $\beta^i \neq 0$, and is attributed to the fact that the set of the background couplings g^i considered in the deformed σ-model at hand is *not a complete one*. In concrete examples [9,14,15] including black hole and other singular backgrounds in string theory, such as colliding branes, one may indeed identify σ-models in which *exactly marginal* deformation operators may be constructed, which however involve either *non-propagating* or, in general, *inaccessible to local scattering experiments* solitonic gravitational degrees of freedom. Truncating the theory to the local degrees of freedom accessible to low-energy experiments, then, defines an *effective non-critical* (Liouville) string theory.

The presence of the non-Hamiltonian terms in (13), proportional to $\beta^i \neq 0$, implies *breaking* of CPT symmetry in target space, due to the fact that, in general, a proper S-matrix cannot be defined in Liouville strings [9]. This stems from a steepest-descent contour over which one defines the Liouville-mode path integral in a σ-model [9,16] (c.f. figure 3). The non-factorisability of the \$-matrix, $\$ \neq SS^{\dagger}$, and hence the ill-definition of a proper scattering amplitude, in such cases is due to ultraviolet divergences in the small-world-sheet-area limit [9]. On the other hand, in the infrared limit, $A \to \infty$, the critical string situation, with a well-defined S-matrix, is recovered. The above observations follow formally from considering infinitesimal world-sheet Weyl transformations of correlation

[2] In σ-model perturbation theory, target-space *canonical* quantization is achieved formally upon summing up world-sheet topologies [9]. It can be shown that the Helmholtz conditions for canonical quantization in Liouville σ-models are preserved upon the identification of time with the Liouville mode.

functions of vertex operators [9]. Let an N-point correlator, $\mathcal{A}_N =< V_1 \dots V_N >$, of a Liouville σ-model, on a world-sheet with metric $\gamma_{\alpha\beta}$, $\alpha, \beta = 1, 2$, and consider an infinitesimal Weyl transformation $\gamma_{\alpha\beta} \to (1 + \sigma(z, \bar{z}))\gamma_{\alpha\beta}$, $|\sigma| << 1$. After some straightforward computations one obtains [9]:

$$\delta_\sigma \mathcal{A}_N \sim \left(\sum_i \Delta_i + \frac{1}{A} \sum_i (\frac{\alpha_i}{\alpha} + \frac{Q}{\alpha}) \right) \mathcal{A}_N \sigma \tag{14}$$

where A is the world-sheet area (in a 'fixed-A' σ-model formalism [8]), Q is the central charge deficit of the non-critical string, and the α's are the gravitational anomalous dimensions of the various couplings corresponding to the vertex operators V_i. Notice that the first term on the right hand side of (14) is independent of the area A, and transforms covariantly under Weyl shifts. This is the term that survives the critical string limit. On the other hand, the second set of terms transform *anomalously*, proportionally to the sum of the Liouville anomalous dimensions and the central charge deficit, and hence are *exclusive* to the Liouville nature of the σ-model. Such terms *vanish* only in the critical case, or in the *infrared* limit $A \to \infty$, where the non-critical string approaches a critical-string *equilibrium* situation [9]. Notice also that due to the presence of these $1/A$-terms, there are ultraviolet world-sheet divergences in the region $A \to 0$, which, result in an ill-defined scattering amplitude. In a critical string theory such A-dependent terms are absent, and hence the correlator has a well-defined meaning as a scattering amplitude. On the other hand, if one defines the Liouville-mode-path-integrated correlator $\mathcal{A}_N$ on the regularizing contour of fig. 3, such divergences are regularized, but, as mentioned earlier, the resulting expression is identified with a \$-matrix element, rather than a scattering amplitude [9]. In this context, the world-sheet divergences may thus be held responsible for the *non factorisability* of the \$-matrix to a product of proper scattering amplitudes.

The lack of this property, implies, then, according to the arguments of [3] reviewed above, a violation of CPT symmetry, at least in its strong form. From (13) we observe that this violation if proportional to β^i. In the cosmological case of the Λ-Universe under consideration the non-criticality is determined primarily by the graviton β-function (8), which in turn implies that the CPT-breaking terms in the evolution equation (13) are of order (in, say, GeV):

$$\text{Cosmological CPT} - \text{breaking in Liouville string} = \mathcal{O}\left((\alpha')^{3/2} \Lambda \right) \tag{15}$$

If one identifies ℓ_s with $1/M_P$, with $M_P \sim 10^{19}$ GeV, the four-dimensional Planck scale, and takes into account the current observational limits of $\Lambda \leq 10^{-123} M_P^4$, then the cosmological CPT violating terms in (15) are of order less than $10^{-123} M_P \simeq 10^{-104}$ GeV. This is too small to be detected in current or immediate future particle physics experiments, such as neutral kaon facilities [17], which are considered as sensitive probes of tests of quantum mechanics [18,19]. However, from our considerations above, it follows that astrophysical observations, either through supernovae [4] or through CMB precision observations [6],

exhibit sensitivity that reach such small scales indirectly, and hence once the presence of a cosmological constant is confirmed in the future, this may be considered also as an indirect observation of an induced cosmological CPT violation, in the sense defined above.

4 Phenomenology of CPT-Violation due to Unitarity breaking

In general, within a quantum-gravity (QG) context, the Lindblad type violation of CPT implied by evolution equations of the form (6), or (13) may be tested in particle physics neutral meson experiments such as neutral kaons [18,19], given that quantum gravity induced *local* violations of unitarity, due to processes involving fluctuating microscopic black holes (c.f. figure 1) may be much larger than the above-mentioned global effects (15). This is due to the fact that such local effects do not necessarily have the time-attenuation factor that characterizes the above-mentioned relaxation models of the cosmological constant. For instance, consider the case of a string propagating in a Black Hole background. As explained in [9], in this case the violation of world-sheet conformal invariance is due to interactions with the foam, as a result of the truncation of the set of background fields to propagating degrees of freedom only.

In other words, the coefficients $C^i_{i_1\dots i_n}$ in (7), which would vanish identically if the set of couplings were complete (exactly marginal deformations), are now different from zero by terms corresponding to massive string states (with masses which are multiples of M_s, the string mass scale). The latter are either non-propagating d.o.f. or solitonic states inaccessible in principle to local scattering experiments, such as global Bohm-Aharonov phases of matter wave functions induced by the quantum gravity singular metric fluctuations [9]. In this case one can expect $C^i_{i_1\dots i_n}$ to be expanded in a power series of $\alpha' k^2$, where k^2 is an (invariant) four-momentum scale, since only closed string scattering amplitudes correspond to gravitational degrees of freedom [10] [3]. Unless prevented by special reasons, which are not expected in general, the series starts from $\alpha' k^2 = k^2/M_s^2$, which in a Lorentz invariant theory is of order m^2/M_s^2, where m a characteristic (rest) mass scale in the problem. This is the order of the β-functions appearing in (13), and hence in such a case one would have:

$$\mathrm{QG-induced\ CPT-breaking\ in\ Liouville\ string} = \mathcal{O}\left(\frac{m^2}{M_s}\right) \tag{16}$$

As mentioned above, the string mass scale M_s may or may not be equal to the four-dimensional Planck mass $M_P \sim 10^{19}$ GeV. Thus, if such a situation is actually encountered in nature, the associated CPT breaking effects are much larger than the cosmological effects (15). This is due to the fact that in these examples the conformal invariance is violated at tree level in a world-sheet genus

[3] If open strings are involved, the power series corresponds to gauge excitations, and the expansion is in terms of $\sqrt{\alpha'}|k|$.

expansion, due to a truncation of the spectrum. If the violation occurs at higher genera, as could be the case of a more conventional violation due to graviton loops in a field-theory context, then one expects in that case a further suppression of (16) by an extra factor m/M_s, i.e. one has CPT-violating terms of order $\mathcal{O}(m^3/M_s^2)$.

Such effects may then be bounded (or tested!) experimentally by taking into account that their presence may induce decoherence and oscillations in, say, neutral mesons [18,19,20,21], neutrinos [22,23] *etc.*. We shall discuss first the neutral meson case, concentrating on the most sensitive probe that of neutral kaons.

The QG induced oscillations are between Kaon and its antiparticle $K^0 \to \overline{K}^0$ [18,19]. The modified evolution equation for the respective density matrices of neutral kaon matter in the *linearized* approximation, (6) or (13), can be parametrized as follows [18]:

$$\partial_t \rho = i[\rho, H] + \delta \not{H} \rho \ ,$$

where

$$H_{\alpha\beta} = \begin{pmatrix} -\Gamma & -\frac{1}{2}\delta\Gamma & -\mathrm{Im}\Gamma_{12} & -\mathrm{Re}\Gamma_{12} \\ -\frac{1}{2}\delta\Gamma & -\Gamma & -2\mathrm{Re}M_{12} & -2\mathrm{Im}M_{12} \\ -\mathrm{Im}\Gamma_{12} & 2\mathrm{Re}M_{12} & -\Gamma & -\delta M \\ -\mathrm{Re}\Gamma_{12} & -2\mathrm{Im}M_{12} & \delta M & -\Gamma \end{pmatrix}$$

and

$$\delta \not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix} \ .$$

Positivity of ρ requires: $\alpha, \gamma > 0, \quad \alpha\gamma > \beta^2$. Notice that α, β, γ violate CPT, as they do not commute with a CPT operator Θ connecting K^0 to $\overline{K}^0$ [19]: $\Theta = \sigma_3 \cos\theta + \sigma_2 \sin\theta, \quad [\delta \not{H}_{\alpha\beta}, \Theta] \neq 0$.

An important remark is now in order. We should distinguish two types of CPT violation (CPTV): (i) CPTV within a Quantum Mechanical formalism: $\delta M = m_{K^0} - m_{\overline{K}^0}$, $\delta\Gamma = \Gamma_{K^0} - \Gamma_{\overline{K}^0}$. This could be due to (spontaneous) Lorentz violation and/or violations of locality (c.f. below).
(ii) CPTV through decoherence α, β, γ (entanglement with QG 'environment', leading to modified evolution for ρ (5) and $\$ \neq S\ S^\dagger$).

The important point is that the two types of CPTV can be *disentangled experimentally* [19]. The relevant observables are defined as $\langle O_i \rangle = \mathrm{Tr}\,[O_i \rho]$. For neutral kaons, one looks at decay asymmetries for $K^0, \overline{K}^0$, defined as:

$$A(t) = \frac{R(\bar{K}^0_{t=0} \to \bar{f}) - R(K^0_{t=0} \to f)}{R(\bar{K}^0_{t=0} \to \bar{f}) + R(K^0_{t=0} \to f)} \ ,$$

where $R(K^0 \to f) \equiv \mathrm{Tr}\,[O_f \rho(t)] =$ denotes the decay rate into the final state f (starting from a pure K^0 state at $t = 0$).

Process	QMV	QM
$A_{2\pi}$	$\neq$	$\neq$
$A_{3\pi}$	$\neq$	$\neq$
$A_{\rm T}$	$\neq$	$=$
$A_{\rm CPT}$	$=$	$\neq$
$A_{\Delta m}$	$\neq$	$=$
ζ	$\neq$	$=$

Table 1. Qualitative comparison of predictions for various observables in CPT-violating theories beyond (QMV) and within (QM) quantum mechanics. Predictions either differ ($\neq$) or agree ($=$) with the results obtained in conventional quantum-mechanical CP violation. Note that these frameworks can be qualitatively distinguished via their predictions for $A_{\rm T}$, $A_{\rm CPT}$, $A_{\Delta m}$, and ζ.

In the case of neutral kaons, one may consider the following set of asymmetries: (i) *identical final states*: $f = \bar{f} = 2\pi$: $A_{2\pi}$, $A_{3\pi}$, (ii) *semileptonic* : A_T (final states $f = \pi^+ l^- \bar{\nu} \neq \bar{f} = \pi^- l^+ \nu$), A_{CPT} ($\overline{f} = \pi^+ l^- \bar{\nu},\ f = \pi^- l^+ \nu$), $A_{\Delta m}$. Typically, for instance when final states are 2π, one has a time evolution of the decay rate $R_{2\pi}$: $R_{2\pi}(t) = c_S\, e^{-\Gamma_S t} + c_L\, e^{-\Gamma_L t} + 2c_I\, e^{-\Gamma t}\cos(\Delta m t - \phi)$, where S=short-lived, L=long-lived, I=interference term, $\Delta m = m_L - m_S$, $\Gamma = \frac{1}{2}(\Gamma_S + \Gamma_L)$. One may define the *Decoherence Parameter* $\zeta = 1 - \frac{c_I}{\sqrt{c_S c_L}}$, as a measure of quantum decoherence induced in the system. For larger sensitivities one can look at this parameter in the presence of a regenerator [19]. In our decoherence scenario, ζ depends primarily on β, hence the best bounds on β can be placed by implementing a regenerator [19].

The experimental tests (decay asymmetries) that can be performed in order to disentangle decoherence from quantum mechanical CPT violating effects are summarized in table 1. In figure 4 we give a typical profile of a decay asymmetry, that of A_T [19], from where bounds on QG decoherening parameters can be extracted. The other asymmetries may be studied in a similar fashion. Details can be given in [19], where we refer the interested reader for details. Experimentally, the best available bounds at present come from CPLEAR measurements [17] $\alpha < 4.0 \times 10^{-17}$ GeV , $|\beta| < 2.3. \times 10^{-19}$ GeV , $\gamma < 3.7 \times 10^{-21}$ GeV, which are not much different from theoretically expected values $\alpha\ ,\beta\ ,\gamma = O(\xi \frac{m_K^2}{M_P})$, where m_K is the Kaon rest mass.

One may extend the above formalism to study correlated Kaon states, as those produced in a ϕ decay [20], $\phi \rightarrow K^0 \overline{K}^0$. It is interesting to note that in such cases the non-quantum mechanical terms in (6) produce terms that apparently violate energy and angular momentum, at a microscopic level, and this is consistent with generic properties of the formalism [19]. It must be stressed though that the formalism remains still an open issue, given that the evolution of correlated states may require genuine two-particle state variables for the non-quantum mechanical parts, whilst in [20] one used only single-particle variables α, β, γ. An additional point is the validity of the requirement of *complete positivity* of the reduced density matrix of the correlated Kaon states [21], which

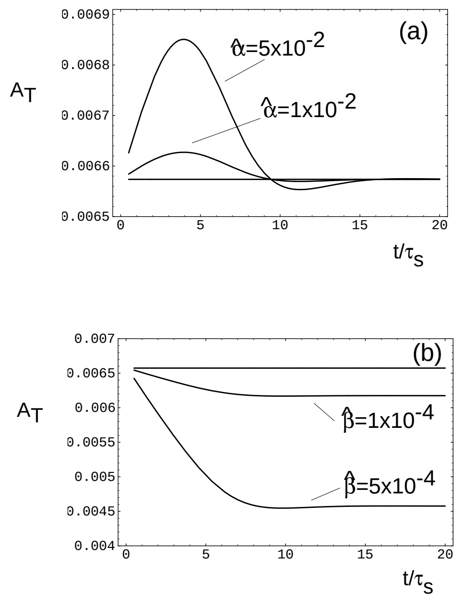

Fig. 4. A typical neutral kaon decay asymmetry A_T [19] indicating the effects of quantum-gravity induced decoherence.

would impose further restrictions on the set of the QG decoherence parameters. However, in view of potential *non-linearities* of quantum gravity, or other mean-field effects, it is not clear whether such a requirement actually holds [24,25]. Such issues present interesting challenges for the theory.

Finally, when the above-described formalism of open quantum systems [18] is applied to neutrinos, induced oscillations between neutrino flavours may occur as a result of decoherence [22,23], which may be independent of neutrino masses. Fitting currently available data from atmospheric and solar neutrinos [22] one may obtain sensitivities which exceed the Planck scale by far. For instance, concentrating on the QG-decoherence-induced transition probability $P_{\nu_e \to \nu_\mu}(t)$, and *assuming* that the dissipative environmental (QG) effects are of order

$$\frac{< E >^2}{M_{QG}} \ , \tag{17}$$

where $< E >$ is an average neutrino energy, the authors of [23] estimated the decoherence effects to be smaller than 10^{-27} GeV, implying an effective QG scale for massless neutrino much larger than M_P for 10^3 GeV energies. On the other hand, an earlier analysis [22] by means of the super-Kamiokande data [26] for $\nu_\mu - \nu_\tau$ oscillations yields a somewhat weaker bound for the QG-decoherence effects $< 3.5 \times 10^{-23}$ GeV in the massive neutrino case, using the super Kamiokande value [26] for the neutrino squared mass difference $\Delta m^2 = m_2^2 - m_1^2 = 3 \times 10^{-3}$

eV^2 between the oscillating states. In the massless neutrino case, the analysis of [22] yields more stringent bounds for the decoherence parameter(s), of order 10^{-27} GeV, as in [23].

However, theoretically, for massive neutrino models of decoherence, more conservative estimates have been presented for *some models* [27] according to which the decoherence parameters are of order

$$\frac{(\Delta m^2)^2}{< E >^2 M_{QG}} \tag{18}$$

which is much more pessimistic than (17), and is probably not detectable at immediate future facilities. The situation is far from being conclusive, however, as it depends on the details of the QG environment and its interaction with the subsystem [25].

A comment we would like to make concerns the presence of energies E in both estimates (17) and (18), which makes them appropriate only for *non Lorentz invariant* formulations. For Lorentz Invariant (LI) cases one would expect only rest mass terms to appear, for instance (18) could be replaced by $\left(\Delta m^2/M_{QG}\right)$ in minimal suppression models *etc.* For *massless* neutrinos the LI formulation is tricky, as one lacks a fundamental low-energy mass scale. Some steps towards LI decoherence are taken in [28], where a LI decoherence has been defined in terms of *intrinsic quantum mechanical uncertainties* of spatial translations, like the ones entering the generalized uncertainty principle in string theory [29]. However, in our opinion, the issue remains a challenging open one.

5 Quantum Gravity, Lorentz violation and CPT

The above discussion assumed that Lorentz invariance is maintained by quantum gravity. This may not be the case in certain backgrounds, for instance those involving *spontaneous breaking of Lorentz symmetry* by means of certain tensor quantities acquiring vacuum expectation values [30], $< A_{\mu_1 \dots \mu_N} > \neq 0$, which may characterize some string theory backgrounds. Another set of models with potential Lorentz violations are loop quantum gravity models [31], as well as approaches to quantum gravity viewing the Planck length as a 'real length' [32], for which the requirement of not being subjected to Lorentz contraction leads to modified dispersion relations for matter probes, including photons, already in flat Minkowski space times. The breaking of Lorentz symmetry violates the requirement (ii) of the CPT theorem stated in the introduction, and hence leads to a breaking of CPT symmetry of the associated field theory.

In such models one may encounter situations in which the violations of Lorentz Invariance (LIV) and, hence, CPT, are *minimally* suppressed by a single power of the Planck mass scale, $M_P \sim 10^{19}$ GeV (or, in general, a QG scale, M_{QG}, which may be different from M_P), and are typically of order

$$\text{Minimally suppressed LIV CPTV} = \mathcal{O}\left(\frac{E^2}{M_P}\right) \tag{19}$$

where E is a typical energy scale of the matter probe, in the frame of observation.

We must note here that as a result of LIV there is a *frame dependence* of the results, and probably a preferred frame, which may be taken to be the CMB frame (but this may not the only possibility).

The basic formalism is described in [30], where one considers *modified Dirac equation (MDE)* for fermions in the so-called Standard Model Extension (SME). In view of the recent 'massive' production of antihydrogen ($\overline{H}$) at CERN [33], which implies that interesting direct tests of CPT invariance using $\overline{H}$ are to be expected in the near future, we consider for our purposes here the specific case of MDE for Hydrogen H (anti-hydrogen $\overline{H}$), although the formalism is generic. Let the spinor ψ represent the electron (positron) with charge $q = -|e| (q = |e|)$ around a proton (antiproton) of charge $-q$. Then the MDE reads:

$$\left(i\gamma^\mu D^\mu - M - a_\mu\gamma^\mu - b_\mu\gamma_5\gamma^\mu - \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu} + ic_{\mu\nu}\gamma^\mu D^\nu + id_{\mu\nu}\gamma_5\gamma^\mu D^\nu\right)\psi = 0,$$

where $D_\mu = \partial_\mu - qA_\mu$, $A_\mu = (-q/4\pi r, 0)$ Coulomb potential. The parameters a_μ , b_μ induce CPT and Lorentz violation, while the parameters $c_{\mu\nu}, d_{\mu\nu}, H_{\mu\nu}$ induce Lorentz violation only.

In SME models there are energy shifts between states $|J, I; m_J, m_I>$, with $J(I)$ denoting electronic (nuclear) angular momenta. Using perturbation theory, one finds [30]:

$$\Delta E^H(m_J, m_I) \simeq a_0^e + a_0^p - c_{00}^e m_e - c_{00}^p m_p + (-b_3^e + d_{30}^e m_e + H_{12}^e)\frac{m_J}{|m_J|} +$$
$$(-b_3^p + d_{30}^p m_p + H_{12}^p)\frac{m_I}{|m_I|} \ ,$$

where e electron; p proton. The corresponding results for antihydrogen ($\overline{H}$) are obtained upon:

$$a_\mu^{e,p} \to -a_\mu^{e,p} \ , \ b_\mu^{e,p} \to -b_\mu^{e,p} \ , \ d_{\mu\nu}^{e,p} \to d_{\mu\nu}^{e,p} \ , H_{\mu\nu}^{e,p} \to H_{\mu\nu}^{e,p} \ .$$

One may study the spectroscopy of *forbidden transitions 1S-2S*: If CPT and Lorentz violating parameters are constant they drop out to leading order energy shifts in free H ($\overline{H}$). Subleading effects are then suppressed by the square of the fine structure constant: $\alpha^2 \sim 5 \times 10^{-5}$, specifically: $\delta\nu_{1S-2S}^H \simeq -\frac{\alpha^2 b_3^e}{8\pi}$. This is too small to be seen.

But what about the case where atoms of H (or $\overline{H}$) are in magnetic traps? Magnetic fields induce hyperfine Zeeman splittings in 1S, 2S states. There are four spin states, mixed under the the magnetic field B ($|m_J, m_I>$ basis): $|d>_n = |\frac{1}{2}, \frac{1}{2}>$, $|c>_n = \sin\theta_n|-\frac{1}{2}, \frac{1}{2}> + \cos\theta_n|\frac{1}{2}, -\frac{1}{2}>$, $|b>_n = |-\frac{1}{2}, -\frac{1}{2}>$, $|a>_n = \cos\theta_n|-\frac{1}{2}, \frac{1}{2}> - \sin\theta_n|\frac{1}{2}, -\frac{1}{2}>$, where $\tan 2\theta_n = (51\mathrm{mT})/\mathrm{n}^3\mathrm{B}$. The $|c>_1 \to |c>_2$ transitions yield dominant effects for CPTV [30]:

$$\delta\nu_c^H \simeq -\frac{\kappa(b_3^e - b_3^p - d_{30}^e m_e + d_{30}^p m_p - H_{12}^e + H_{12}^p)}{2\pi} \ ,$$

LEADING ORDER BOUNDS

EXPER.	SECTOR	PARAMS. (J=X,Y)	BOUND (GeV)
Penning Trap	electron	$\bar{b}_J^e$	5×10^{-25}
Hg−Cs clock comparison	electron	$\bar{b}_J^e$	10^{-27}
	proton	$\bar{b}_J^p$	10^{-27}
	neutron	$\bar{b}_J^n$	10^{-30}
H Maser	electron	$\bar{b}_J^e$	10^{-27}
	proton	$\bar{b}_J^p$	10^{-27}
spin polarized matter	electron	$\bar{b}_J^e$ / $\bar{b}_Z^e$	10^{-29} / 10^{-28}
He−Xe Maser	neutron	$\bar{b}_J^n$	10^{-31}
Muonium	muon	$\bar{b}_J^\mu$	2×10^{-23}
Muon g−2	muon	$\bar{b}_J^\mu$	5×10^{-25} (estimated)

X,Y.Z celestial equatorial coordinates $\bar{b}_J = b_3 - md_{30} - H_{12}$

(Bluhm, hep−ph/0111323)

Fig. 5. Table summarizing recent bounds of CPT violating parameter b in the Standard Model extension from atomic and nuclear physics spectroscopic tests [36].

$$\delta\nu_c^{\overline{H}} \simeq -\frac{\kappa(-b_3^e + b_3^p - d_{30}^e m_e - d_{30}^p m_p - H_{12}^e + H_{12}^p)}{2\pi} ,$$

$$\Delta\nu_{1S-2S,c} \equiv \delta\nu_c^H - \delta\nu_c^{\overline{H}} \simeq -\frac{\kappa(b_3^e - b_3^p)}{\pi} ,$$

where $\kappa = \cos 2\theta_2 - \cos 2\theta_1$, $\kappa \simeq 0.67$ for $B = 0.011$ T. Notice that $\Delta\nu_{c\to d} \simeq -2b_3^p/\pi$, and, *if a frequency resolution of 1 mHz is attained*, one may obtain a bound $|b_3| \leq 10^{-27}$GeV .

The existence of a preferred frame implies very stringent restrictions on such LIV and CPTV terms, most of which stem from observations (both terrestrial and astrophysical) pertaining to *electrically charged* fermions [34,35]. These observations seem to exclude QG-induced *minimally suppressed (linear)* modifications of matter dispersion relations, given that the sensitivity of such experiments exceeds the Planck scale 10^{19} GeV by several orders of magnitude, and hence such models are excluded on naturalness grounds.

For instance, one of the most stringent constraints at present on linearly modified electron dispersion relations is obtained from observations of the synchrotron radiation from Crab Nebula [35], with sensitivity that exceeds $M_P \sim$

10^{19} GeV by nine orders of magnitude. Also, one may obtain high sensitivities in atomic and nuclear physics experiments [34]. A summary of the various sensitivities from atomic physics experiments is given in the table of figure 5 [36], where we can see that in some cases CPT and LIV violating terms for electrons (or electrically charged fermions) may reach sensitivities up to 10^{-31} GeV !

6 Non-Critical String Models of Foam, the Equivalence principle and the Evasion (?) of the Constraints

Although from the above discussion it seems that current experiments exhibit sensitivities that exclude linearly modified dispersion relations, however the actual situation may be more subtle. In fact, we shall argue below that in a Liouville string approach to foam, there is a *violation* of the *equivalence principle* in the sense that not all matter probes interact the same way with the gravitational foamy backgrounds. In particular, photons, or at most particles *neutral* under the (unbroken) standard model gauge group $SU(3)_c \otimes U(1)$, are the only types that may exhibit modified dispersion relations [37]. The rest of the low-energy modes are insensitive to the quantum gravity foamy effects, as far as dispersion relations are concerned.

A concrete model of Liouville string foam with this property has been presented in [38]. According to this model our world is a three brane, and the observable matter particles (including radiation) are viewed as open string excitations on it, with their ends attached to the brane. The brane is embedded in a higher (bulk) space time in which only closed strings (gravitational d.o.f.) are allowed to propagate. The foam is obtained by assuming quantum fluctuations of the brane world which are such that (virtual) D-particle defects can emerge from the brane. These are short-lived excitations, with average life time $\tau \sim \frac{\ell_s}{c}$ (where ℓ_s is the string length, and c the speed of light in empty space). The D-particles are degenerate forms (point-like) of D-branes [39], but as their higher-dimensional counterparts they can *capture* open strings, since the latter may have their ends attached to the D-particle. From a conformal field theory point of view the capture stage may be described by the following σ-model deformations [40]

$$\oint_{\partial\Sigma} u_i X^0 \Theta_\epsilon(X^0)\partial_n X^i \ , \quad \oint_{\partial\Sigma} \epsilon y_i \Theta_\epsilon(X^0)\partial_n X^i \ , \tag{20}$$

where y_i denotes the location of the D-particle on the D3 brane world, and $u_i = g_s \frac{k_1+k_2}{M_s}$, denotes the recoil velocity (proportional to the momentum transfer), with g_s the string coupling, and M_s the string mass scale. The parameter $\epsilon \to 0^+$ regulates the Heaviside operator $\Theta(X^0)$. It can be shown that the pair of deformations (20) closes to form a *logarithmic conformal* algebra (LCFT), provided ϵ^{-2} is identified with the logarithm of the world-sheet RG scale. Such LCFT lie in the border line between conformal algebras and general two-dimensional field theories. The operators (20) are *marginally relevant* in a

world-sheet RG sense, with anomalous dimensions ϵ^2. This has important implications for the departure of the deformed σ-model from the conformal point, hence the need for Liouville dressing [8].

This dressing results in the appearance of non trivial metric deformations in space time, which physically are interpreted as a back reaction of the recoiling D-particle, during the capture stage, onto the neighboring space time. The resulting metric assumes the form (asymptotically in time after the capture stage) [40]:

$$G_{00} = -1 \ , \qquad G_{ij} = \delta_{ij} \ , \qquad G_{0i} = u_i \tag{21}$$

This change in the space time background, during every interaction of the matter probe with the D-particle, causes a *modification* in the dispersion relation of the probe, as a result of the relation:

$$p_\mu p_\nu G^{\mu\nu} = -m^2 \ \rightarrow \ -E^2 + \boldsymbol{p}^2 + 2E\boldsymbol{p}\cdot\boldsymbol{u} = -m^2(1+\boldsymbol{u}^2) \tag{22}$$

It should be stressed that this modification is consistent with the general coordinate diffeomorphism invariance of the target space of the stringy σ-model, which is respected by the foam. From (22) it is evident that in this way one obtains *linear* modifications in the dispersion relations, and hence such models, if valid for electrically charged fermions, would have been excluded in the sense of yielding $M_s/g_s > 10^{29}$ GeV. We stress that only the *subluminal* branch of the modifications in the dispersion relation is consistent with the capture stage [37], and in fact it is only such modifications that are allowed by the string dynamics [38].

Mathematically, of course, one could still save such models by adjusting M_s or g_s appropriately so as to meet such sensitivities, but then one probably faces a naturalness problem, in that one should explain why this particular model of foam is characterized by such small g_s (assuming $M_s \sim M_P$).

Fortunately for the model of [38] there seems to be a *way out*, which in fact is based on *gauge symmetry principles*. The point is that the capture of an open string by a D-particle has been shown in [40] to describe $U(1)$ gauge excitations, whose dynamics is described by a Born-Infeld (BI) effective Lagrangian $\mathcal{L}_{BI} = \sqrt{G_{\mu\nu} + \alpha' F_{\mu\nu}}$, with $F_{\mu\nu}$ the $U(1)$ Maxwell field strength, and $G_{\mu\nu}$ the background metric. In general, N coincident D-particles exchanging open strings stretched between them, will transform according to the adjoint representation of the $U(N)$ group, and obey a non Abelian BI action [39]. This includes also the recoil case [40]. Thus we conclude that the capture/recoil of open strings from a group of N D-particle excitations in the foam will result in gauge excitations on the D3 brane world. There is no way of obtaining other excitations, transforming for instance according to the fundamental representation of the group. For the latter to happen one needs *intersecting* brane configurations at angles θ [41], the latter serving to define appropriate chirality for fermionic excitations. D-particles are by definition parallel among themselves, and one cannot thus get chiral fermions by exchanging strings among them, a process characterizing the capture/recoil case.

Given that in a space-time foam model the quantum numbers of the ‘vacuum’ must be preserved [42] by the interactions of matter with the foam d.o.f., we must

conclude from the above considerations that only string modes which transform in the adjoint representation of the (unbroken) standard model group $U(3) \simeq SU(2) \otimes U(1)$, and are therefore gauge excitations, can interact in the above sense with the D-particles and have modified dispersion relations of the type (22).

The above result holds at *tree level* in a world-sheet formalism. From a world sheet view point the capture of the open string or closed string by a D-particle implies either changes in the Boundary conditions of the open string (from Neumann (N) to Dirichlet (D) [39]), or splitting of the closed string to a pair of open ones, with Dirichlet boundary conditions. A natural question arises in connection with higher world-sheet topologies, for instance one loop (annulus) world-sheet graphs. Such loops may express, for instance, self energy parts of electrons. Such quantum corrections involve propagating photons (and other (actually, an infinity of) string excitations), and one may expect that these will interact with the D-particles of the foam. The issue as to the precise effects of these loop calculations on the self energy of the electrons and other charged fermions is under investigation. This may also depend on the complicated dynamics of the (yet unknown) M-theory that describes such defects. Such issues deserve further study before conclusions are reached.

The above considerations exclude electrons, and other electrically charged fermions (e.g. quarks) from being captured by the D-particle (at tree level at least), given that the capture of such excitations by the D-particle would lead to violations of electric charge [37]. Thus, modulo higher order string-loop effects, the most stringent constraints on the linear modifications of the dispersion relations, which are based on such fermions [34,35], are evaded in the Liouville string model of [38]. On the other hand, *photons* (and probably gluons [4]), *can* interact with D-particles *at tree level in σ-model perturbation theory*, thereby exhibiting linearly modified dispersion relations and *subluminal* refractive indices (the subluminal nature is due to the Born-Infeld electrodynamics [38]).

Such effects can be tested by γ-Ray Burst studies (GRB) of the arrival times of photons, as suggested in [43]. The sensitivity of such experiments, at present, is such that the QG effective scale for photons exhibiting linear dispersion relations (22) is $M_{QG} > 10^{15}$ GeV [44]. In the model of [38], the interaction of a photon excitation with a D-particle is accompanied by a *random phase* [37] of the re-emitted photon after the capture stage. Such an effect also invalidates claims made in [45] for excluding linear modifications in the photon dispersion relations [5].

---

[4] Confinement of gluons makes their interaction with the foam subtle. In this review we simply assume that gluons can interact like photons with the D-particles, and do not discuss the matter further. We shall come back to such issues in a forthcoming publication.

[5] Apart from such random phases, we also note that the analysis of [45] overestimated the effects of the foam by a huge factor, as argued in [46], which by itself invalidates their conclusions.

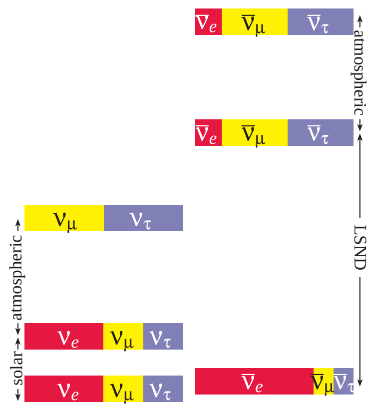

Fig. 6. In the scenario of [50], the LSND anomaly may be explained by a CPTV mass difference between neutrinos and antineutrinos.

It is evident from the above considerations that *details* in the *dynamics* of *space-time foam* do *matter* in discussing the pertinent phenomenology. Many conclusions, based on generic arguments from our experience so far with ordinary local quantum field theories, may be misleading when applied to quantum gravity.

7 CPT Breaking through Locality Violation and Neutrino Anomalies(?)

A final topic we would like to discuss briefly is the violation of CPT in the neutrino sector of the theory. The reason why we chose to discuss this topic in a separate section is the relatively recent claims from the LSND experiment [47] on evidence for observation of oscillations in the antineutrino sector $\overline{\nu}_e \longleftrightarrow \overline{\nu}_\mu$, but the absence of such oscillation in the corresponding neutrino sector [6]. The situation will be clarified experimentally, when experiments looking directly for $\nu_\mu \to \nu_e$, will be in operation, like MiniBoonNE [48] [7].

If one associates oscillations with Dirac mass terms for neutrinos, then the LSND result [47], if correct, points towards CPT violation in the sense of an

[6] The initial 2.6σ hint for $\nu_\mu - \nu_e$ decreased to 0.6σ, while the signal for antineutrino oscillations remained.

[7] The material of this review was presented and written before the very recent announcement on September 7 2003 by the Sadbury Neutrino Observatory (SNO), on improved measurements of neutral current events, which, together with other existing data, confirm the three-neutrino scenaria. The phenomenology presented in this section is before these results. For a status of the three neutrino oscillations after the latest SNO results we refer the reader to [49]. The ideas on CPT violation presented here may then be tested by such updated data.

antineutrino Dirac mass being much higher than a neutrino one, the mass difference between neutrino and antineutrino being of the order of 1 eV. This idea was put forward first in [50] in a purely phenomenological setting (c.f. fig. 6), in an attempt to explain the LSND anomaly [47] without invoking a *sterile* neutrino.

There are many microscopic theories which could lead to such violations. One obvious one is the spontaneous violation of Lorentz symmetry, along the lines of the SME [30], described briefly above, whose applications to the massive neutrino physics has been discussed recently in [51].

On the other hand, in the Liouville model of D-particle foam of [38], neutrinos may not have modified dispersion relations, as their capture stage by D-particles would not seem to respect the gauge quantum numbers of the vacuum. We remind the reader that neutrinos, viewed as open string excitations on the brane, transform in the fundamental representation of appropriate gauge groups, while a string-D-particle 'composite', describing the capture stage, behaves as a gauge field excitation in the model, transforming in the adjoint representation of the unbroken standard model group [8]. However, in other models of QG, where gravitational fluctuations imply the existence of an 'environment' [31,22] neutrinos may have non trivial refractive indices (for the massless species). Such modifications, as we discussed in previous sections, may lead to CPTV and LIV effects of order $g_s E^2/M_s$, where E is a typical neutrino energy at the frame of observation. Such terms seem to be much smaller than the suggested neutrino-antineutrino squared mass difference of $\mathcal{O}(0.1 - 1~\mathrm{eV}^2)$ to explain the LSND anomaly [47,50].

A more radical approach has been suggested earlier in [52], according to which the neutrino sector of the standard model exhibits *non local* interactions among the neutrinos, responsible for the generation of CPT Violating but Lorentz invariant mass spectra for neutrinos, fitting phenomenologically the LSND results. Specifically, the model invokes a Dirac-like theory of neutrinos (called "homeotic") with *both* positive (+) and negative (-) energies, in which the spinors are described by:

$$
\begin{aligned}
&\psi_+(x) = u_+(p)e^{-ip\cdot x}, \quad p^2 = m^2,~ p_0 > 0 \\
&\psi_-(x) = u_-(p)e^{-ip\cdot x}, \quad p^2 = m^2,~ p_0 < 0 \\
&(p_\mu \gamma^\mu - m\epsilon(p_0))u_\pm(p) = 0 ~,
\end{aligned} \tag{23}
$$

where ψ are the fermionic *Dirac* neutrino fields, and $\epsilon(p_0)$ is a sign function. The corresponding part of the effective action responsible for the generation of

[8] Nevertheless, one may envisage the possibility of neutrinos interacting with D-particle defects in brane models intersecting at angles [41], where chiral fermions are open string excitations localised on the intersection hypersurface, viewed as our world. We do not know yet whether puncturing the intersection with D-particles can be consistent with ND chiral open strings with mixed boundary conditions, corresponding to neutrinos, with one end attached to the D-particle (D), and the other end free (N) on the intersection, that would allow neutrinos (as electrically neutral) to have modified dispersion relations.

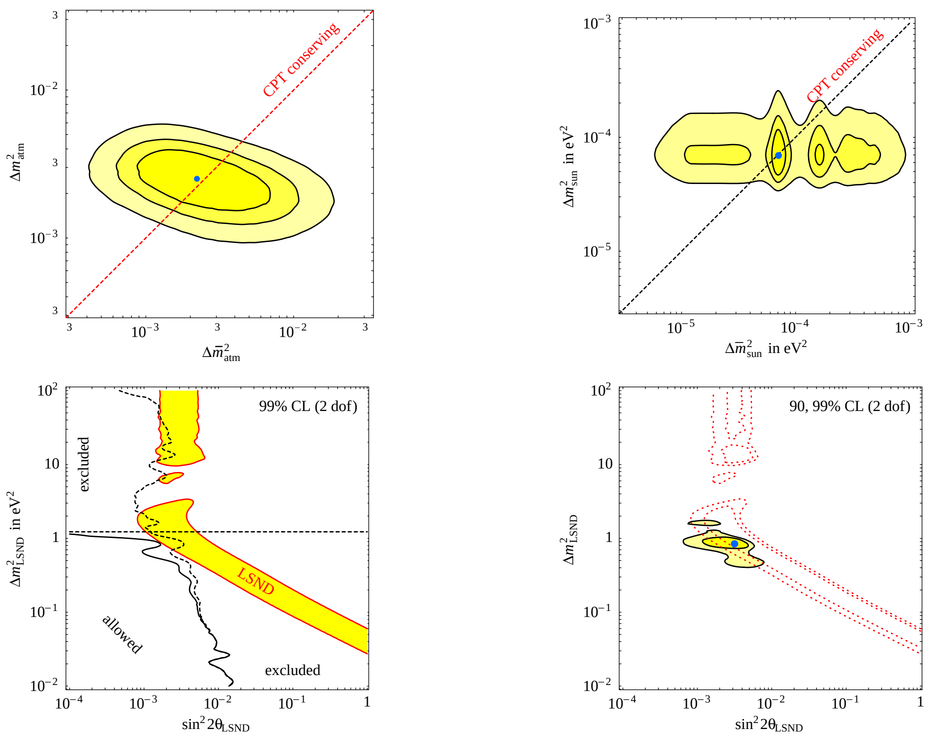

Fig. 7. The phenomenology of neutrino models involving CPT Violating mass spectra done in [53] seems to be disfavored marginally when all the available current data are compiled. Upper figures: (a) Left: Atmospheric $m_\nu - m_{\overline{\nu}}$ (68, 90, 99 %, 2 d.o.f.). (b) Right: For solar and reactor data (68, 90, 99 %, 2 d.o.f.). Lower Figures: (a) Left: Upper half plane disfavored by WMAP. Dashed curved line: upper bound from all other ν experiments. (b)Right: Best fit, all data; 3 + 1 sterile ν solution disfavored by WMAP [6], since $(\Delta m^2_{LSND})^{1/2} \simeq \sum m_\nu$. *Notation*: The oscillation probabilities are: $P_{\nu_e \to \nu_e} = 1 - S\sin^2\theta_{es}$, $P_{\nu_\mu \to \nu_\mu} = 1 - S\sin^2\theta_{\mu s}$, $P_{\nu_e \to \nu_\mu} = S\sin^2\theta_{LSND}$, with $\sin^2\theta_{LSND} \simeq \frac{1}{4}\sin^2 2\theta_{es}\sin^2 2\theta_{\mu s}$.

a CPTV mass spectrum according to this scenario reads:

$$S = \int d^4x \bar{\psi} i \partial_\mu \gamma^\mu \psi + \frac{im}{2\pi} \int d^3x dt dt' \bar{\psi}(t) \frac{1}{t - t'} \psi(t') \qquad (24)$$

Notice that in (24) Lorentz invariance is maintained (at least at tree level) due to the presence of the $\epsilon(p_0)$. However *Locality* is relaxed, and hence CPT. We stress once again that in this scenario the neutrino masses are of *Dirac* type. An open issue, of course, of such scenaria for neutrino CPTV mass spectra is what singles out the neutrino sector from the rest of the standard model so as to produce mass differences of the order of the LSND anomaly.

The phenomenology of the model has been analyzed in [53], where it was argued that a compilation of all available data from current neutrino experiments does not seem to favor CPT Violating scenaria for neutrino mass spectra. The point is that if CPT is Violated in the neutrino sector, then such violations will imply signals in atmospheric and solar ν oscillations. The analysis of [53] combined recent results from KamLAND [54] with atmospheric data, as well as recent WMAP satellite data [6]. The results of such an analysis are given in fig. 7, from where it is evident that a CPTV ν-mass scenario is excluded. However, the conclusion is marginal, as can be seen from the figure, and moreover it pertains only to CPT violation within conventional quantum mechanics, in the sense of a violation being realized through a neutrino-antineutrino mass difference.

As we have discussed above, QG effects of the type appearing in the non-quantum mechanical evolution equations (5) or (6),(13) may be themselves responsible for neutrino oscillations, in a way independent of mass terms [22,23]. Since such models violate CPT through the ill-definition of a scattering matrix, as explained above, they may provide a natural explanation for the LSND anomaly, provided the latter is confirmed by future experiments. If this is the case, phenomenological analyses such as that in [53] have to be redone by taking into account decoherence effects in the dynamics of neutrinos.

Determining the order of such CPTV effects in the neutrino sector is not an easy task, and is highly model dependent. However, if QG is responsible for the effects only through unitarity violation, and Lorentz invariance and locality are preserved, we expect that the order is (more or less) universal among all particle species. As already mentioned, stringent bounds on such non quantum mechanical QG-induced decoherence parameters have been derived for the neutrino sector [22,23], which are much smaller than the corresponding bounds in the Kaon sector [17]. This may guide us in theoretical modelling of these effects.

8 Conclusions and Outlook

In this brief review we considered some models of CPT violation and their associated phenomenological constraints. From our discussion it becomes clear that in estimating the order of the effects, and hence the sensitivity of the various experiments, it is important to know the details of the gravitational environment that may be responsible for such violations. There is no single figure of merit for CPT violation, and hence detailed and systematic analyses have to be performed with care before conclusions are reached.

For instance, according to some non-critical stringy models of foam, the photons (and probably gluons) may exhibit modified dispersion relations, and hence LIV and CPTV, but such properties *may not* characterize the rest of the particles in the standard model. This implies that future experiments testing photon dispersion relations are important in shedding light in the quantum nature of space time.

It may well be, of course, that CPT is broken, if at all, only cosmologically, in the sense discussed in the beginning of the article, in which case any direct test

via particle physics experiments seems pointless due to the very small value of the associated effects. However, such effects may be tested indirectly by means of astrophysical observations, for instance experiments associated with a direct measurement of the acceleration of the Universe [4], or measurements of the CMB anisotropies to a very high precision [6]. It is fascinating to link a possible confirmation of a cosmological *constant* in our Universe with a breaking of CPT symmetry by a tiny but finite amount. Time will tell whether CPT symmetry is sacrosanct or follows the fate of so many other symmetries in nature, being broken by quantum space-time effects. Fortunately such a question may be tackled by many experiments in the foreseeable future, especially those from the astrophysics side.

Astrophysics has made enormous progress in improving the experimental sensitivities over the past few years, which allows tiny numbers, such as the cosmological constant (if the latter is non zero), to be measured directly! Thus, a fruitful co-operation with Particle Physics is to be expected for the exciting years to come. In this review, we made an attempt to associate astrophysical/cosmological phenomena, such as the Dark Energy content of the Universe, to fundamental concepts of Quantum Field Theory, such as CPT (non)invariance, and the structure of quantum space time. This may not be the only example, where such a connection exists. Neutrinos, as we well as Supersymmetry searches, which we did not discuss here, constitute topics where new physics would definitely come into play, and where astrophysics has already given important results and/or constraints [6,55]. One cannot also exclude the possibility of having pleasant surprises from antimatter factories, such as [33], which may prove to be important for precision tests of CPT symmetry in the not-so-distant future.

One therefore expects a plethora of experiments from both Particle and Astrophysics side in the next decade at least, which hopefully will provide us with interesting physical results, and thus enable us to make several steps forward in our quest for understanding the fundamental interactions in Nature. I would like to close this discussion with a remark made by Okun in a lecture on CPT symmetry [56]: if CPT is broken, he said, then the whole structure on which we built the current form of quantum field theory, and on which our phenomenology is based, may cease to exist. How can we proceed then, so as to make sure that we detect and interpret such a violation correctly? As we have discussed in this review, this may indeed turn out to be true, but it may not be so drastic as one thinks. The dynamics of open systems, for instance, familiar from other fields of physics, such as condensed matter, may be the way forward. Time will tell...

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